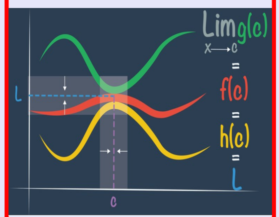
Calculus I

Lecture 15



Feb 19-8:47 AM

Class Quiz 15

Given $x^2 + xy + y^2 = 1$

Evaluate $\boxed{\frac{dy}{dx} \text{ at } (1,-1).}$ $\quad \left.\frac{dy}{dx}\right|_{(1,-1)}$

$$\frac{d}{dx}\left[x^2 + xy + y^2\right] = \frac{d}{dx}[1]$$

$$\frac{d}{dx}[x^2] + \frac{d}{dx}[xy] + \frac{d}{dx}[y^2] = 0$$

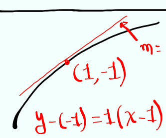
$$2x + 1\cdot y + x\cdot\frac{dy}{dx} + 2y\cdot\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$$

$$(x + 2y)\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$$

$$\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{-2(1) - (-1)}{1 + 2(-1)} = \boxed{1}$$

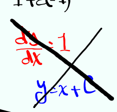


m = 1

(1, -1)

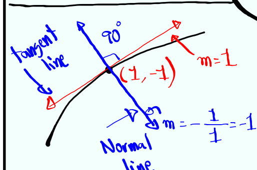
$y - (-1) = 1(x - 1)$

$\boxed{y = x - 2}$



$\frac{dy}{dx} = 1$

$y = x + C$



tangent line

90°

(1, -1)

m = 1

$m = -\frac{1}{1} = -1$

Normal line

$y - (-1) = -1(x - 1)$

$y + 1 = -x + 1$

$\boxed{y = -x}$

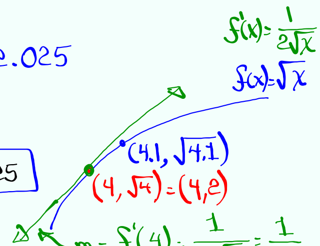
Apr 14-7:47 AM

Evaluate $\sqrt{4.1}$

Using Calc. $\sqrt{4.1} \approx 2.025$

Let $f(x) = \sqrt{x}$

$f(4.1) = \sqrt{4.1} \approx \boxed{2.025}$



$m = f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

Linear Approximation

$f(x) \approx f(a) + f'(a)(x-a)$

$y - 2 = \frac{1}{4}(x-4)$

$y - 2 = \frac{1}{4}x - 1$

$\boxed{y = \frac{1}{4}x + 1}$

at $x = 4$

$y = 2$

$f(x) = \sqrt{x}$ $\quad f(4) = \sqrt{4} = 2$

$a = 4$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$\sqrt{x} \approx 2 + \frac{1}{4}(x-4)$

$\sqrt{4.1} \approx 2 + \frac{1}{4}(4.1-4)$

$= 2 + \frac{1}{4}(.1)$

$= 2 + \frac{1}{40} = \frac{81}{40}$

$\sqrt{4.1} \approx \frac{81}{40}$

$= \boxed{2.025}$

Apr 14-9:12 AM

use linear appoximation to evaluate $\sqrt[3]{27.1}$.

$\sqrt[3]{27.1} \approx \sqrt[3]{27} \approx 3$ $\quad$ Let $f(x) = \sqrt[3]{x}$

$a = 27$

use Calc.

$\sqrt[3]{27.1} \approx \boxed{3.004}$ $\quad f(x) \approx f(a) + f'(a)(x-a)$

$f(x) = \sqrt[3]{x}$ $\quad f(x) = x^{\frac{1}{3}}$

$a = 27$ $\quad f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$

$f(27) = \sqrt[3]{27} = 3$ $\quad f'(27) = \frac{1}{3\sqrt[3]{27^2}} = \frac{1}{3 \cdot 9} = \frac{1}{27}$

$\sqrt[3]{x} \approx 3 + \frac{1}{27}(x-27)$

$\sqrt[3]{27.1} \approx 3 + \frac{1}{27}(27.1-27)$

$\approx 3 + \frac{1}{27}(.1)$

$= 3 + \frac{1}{270}$

$= \boxed{\frac{811}{270}}$

$\sqrt[3]{27.1} \approx \frac{811}{270}$

$\approx \boxed{3.004}$

Apr 14-9:22 AM

Use linear approximation to evaluate $(10.01)^3$

$(10.01)^3 \approx 10^3 = 1000$

$f(x) = x^3$ $\quad f(10) = 1000$

$a = 10$ $\quad f'(x) = 3x^2$

$f'(10) = 3(10)^2 = 300$

using Calc

$10.01^3 = 1003.003$

Linear approximation $f(x) \approx f(a) + f'(a)(x-a)$

$x^3 \approx 1000 + 300(x - 10)$

$(10.01)^3 \approx 1000 + 300(10.01 - 10)$

$= 1000 + 300(.01)$

$= 1000 + 3$

$= 1003$

Apr 14-9:32 AM

Use linear appoximation to evaluate $\sin 88°$.

$\sin 88° \approx \sin 90° = 1$

using Calc

$\sin 88° \approx .999$

$f(x) = \sin x$

Linear Approximation

$a = 90° = \frac{\pi}{2}$

$f(x) \approx f(a) + f'(a)(x - a)$

$f(90°) = \sin 90° = 1$

$\sin x \approx 1 + 0(x - \frac{\pi}{2})$

$f'(x) = \cos x$

$f'(90°) = \cos 90° = 0$

$\sin 88° \approx 1 + 0(88° - 90°)$

$\approx 1 + \cancel{0 \cdot 2°} \rightarrow 0$

$\sin 88° \approx 1$

Apr 14-9:42 AM

Use linear approximation to evaluate $\tan 46^\circ$.

$\tan 46^\circ \approx \tan 45^\circ \approx 1$

Using Calc.

$\tan 46^\circ \approx \boxed{1.036}$

$f(x) = \tan x$ $\quad f(45^\circ) = 1$

$a = 45^\circ$ $\quad f'(x) = \sec^2 x$

$f'(45^\circ) = \sec^2 45^\circ$

$= (\sqrt{2})^2 = 2$

$45^\circ = \frac{\pi}{4}$

$f(x) \approx f(a) + f'(a)(x-a)$

$\tan x \approx 1 + 2\left(x - \frac{\pi}{4}\right)$

$\tan 46^\circ \approx 1 + 2(46^\circ - 45^\circ)$

$180^\circ = \pi$ Rad.

$1^\circ = \frac{\pi}{180}$ Rad.

$= 1 + 2 \cdot 1^\circ$

$= 1 + 2 \cdot \frac{\pi}{180} = 1 + \frac{\pi}{90} \approx \boxed{1.035}$

Apr 14-9:49 AM

Use linear approximation to estimate $\cos 62^\circ$.

Using Calc.

$\cos 62^\circ \approx .469$

$\cos 62^\circ \approx \cos 60^\circ = .5$

$a = 60^\circ \to \frac{\pi}{3}$

$f(x) = \cos x$

$f(a) = f(60^\circ) = \cos 60^\circ = .5$

$f'(x) = -\sin x$

$f'(a) = f'(60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

$f(x) \approx f(a) + f'(a)(x-a)$

$1^\circ = \frac{\pi}{180}$ Rad

$\cos x \approx .5 - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)$

$2^\circ = \frac{\pi}{90}$ Rad

$\cos 62^\circ \approx .5 - \frac{\sqrt{3}}{2}(62^\circ - 60^\circ)$

$= .5 - \frac{\sqrt{3}}{2}(2^\circ)$

$= .5 - \frac{\sqrt{3}}{2} \cdot \frac{\pi}{90} = \frac{1}{2} - \frac{\pi\sqrt{3}}{180} = \boxed{\frac{90 - \pi\sqrt{3}}{180}}$

$\approx \boxed{.470}$

Apr 14-10:01 AM

Suppose $x = x(t)$ and $y = y(t)$ and

$$x^2 + y^2 = z^2 \quad \text{so } z = z(t)$$

$$\frac{d}{dt}\left[x^2 + y^2\right] = \frac{d}{dt}\left[z^2\right]$$

$$\frac{d}{dt}\left[x^2\right] + \frac{d}{dt}\left[y^2\right] = \frac{d}{dt}\left[z^2\right]$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}}{\cancel{2}z}$$

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

Apr 14-10:13 AM

$A = \pi r^2$, $A = A(t)$, and $r = r(t)$

find $\frac{dr}{dt}$

$$\frac{d}{dt}\left[A\right] = \frac{d}{dt}\left[\pi r^2\right]$$

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{\frac{dA}{dt}}{2\pi r}$$

Apr 14-10:18 AM

$V = \frac{4\pi r^3}{3}$, $\boxed{\frac{dV}{dt} = 2\pi \text{ when } r = 5}$ $V = V(t)$ $r = r(t)$

Find $\frac{dr}{dt}$ when $\boxed{r = 5}$

$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4\pi r^3}{3}\right]$

$\frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$2\pi = 4\pi \cdot 5^2 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{\overset{1}{\cancel{2\pi}}}{\underset{2}{\cancel{4\pi}} \cdot 25}$

$\frac{dr}{dt} = \frac{1}{50}$

Apr 14-10:24 AM

$x = x(t)$, $y = y(t)$, $x^2 + y^2 = z^2$

$8^2 + 6^2 = z^2$

$100 = z^2$

$\boxed{z = 10}$

$\frac{dx}{dt} = 6$, $\frac{dy}{dt} = -3$, Find $\frac{dz}{dt}$ when $x = 8$, $y = 6$.

$\frac{d}{dt}[x^2 + y^2] = \frac{d}{dt}[z^2]$

$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

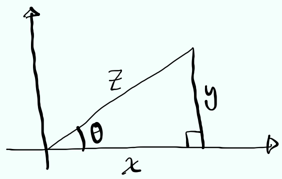
$8 \cdot 6 + 6 \cdot (-3) = 10 \cdot \frac{dz}{dt}$

$48 - 18 = 10\frac{dz}{dt}$

$\frac{30}{10} = \frac{dz}{dt}$ $\boxed{\frac{dz}{dt} = 3}$

Apr 14-10:29 AM

Consider the drawing below



$x = x(t)$

$y = y(t)$

Find $\frac{d\theta}{dt}$ when $x = 4$, $y = 5$

$\frac{dx}{dt} = 2$, $\frac{dy}{dt} = 4$.

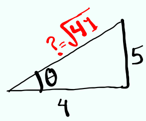
$\sin\theta = \frac{y}{z}$?

$\cos\theta = \frac{x}{z}$?

$\tan\theta = \frac{y}{x}$

$\frac{d}{dt}\left[\tan\theta\right] = \frac{d}{dt}\left[\frac{y}{x}\right]$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{\frac{dy}{dt}\cdot x - y\cdot\frac{dx}{dt}}{x^2}$



$\cos\theta = \frac{4}{\sqrt{41}}$

$\sec\theta = \frac{\sqrt{41}}{4}$

$\sec^2\theta = \frac{41}{16}$

$16 - 10$

$\frac{41}{\cancel{16}} \cdot \frac{d\theta}{dt} = \frac{4\cdot 4 - 5\cdot 2}{\cancel{4^2}}$

$41 \frac{d\theta}{dt} = 6$

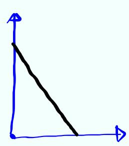
$\frac{d\theta}{dt} = \frac{6}{41}$

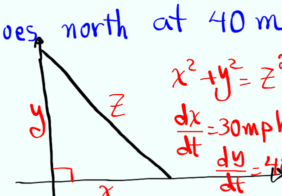
Apr 14-10:39 AM

Two cars are at an intersection.

one goes east at 30 mph.

the other one goes north at 40 mph.

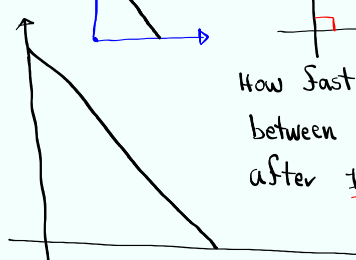




$x^2 + y^2 = z^2$

$\frac{dx}{dt} = 30\text{mph}$

$\frac{dy}{dt} = 40\text{mph}$



How fast is the distance between them changing $\frac{dz}{dt}$ after 1 hr?

$x = 30$ miles

$y = 40$ miles

$x^2 + y^2 = z^2 \longrightarrow 30^2 + 40^2 = z^2$

$2500 = z^2$

$\boxed{z = 50}$

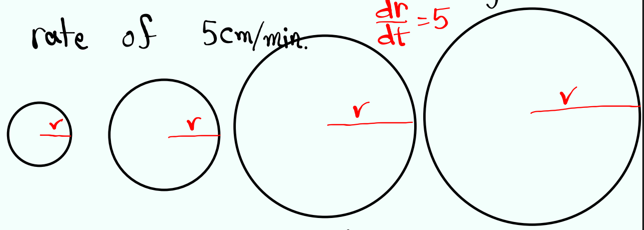
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$30\cdot 30 + 40\cdot 40 = 50\cdot\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{2500}{50} = 50$ mph

Apr 14-10:52 AM

Radius of a circle is increasing at the rate of 5cm/min. $\frac{dr}{dt} = 5$



How fast its area changing when r=8cm.

Area of Circle

$$A = \pi r^2$$

$$\frac{dA}{dt} = \frac{d}{dt}\left[\pi r^2\right]$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi \cdot 8 \cdot 5$$

$$= 80\pi \boxed{cm^2/min.}$$

Related Rates

Apr 14-11:02 AM